A POMDP Approach to P300-Based Brain-Computer Interfaces

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ABSTRACT
Most of the previous work on non-invasive brain-computer interfaces (BCIs) has been focused on feature extraction and classification algorithms to achieve high performance for the communication between the brain and the computer. While significant progress has been made in the lower layer of the BCI system, the issues in the higher layer have not been sufficiently addressed. Existing P300-based BCI systems, for example the P300 speller, use a random order of stimulus sequence for eliciting P300 signal for identifying users’ intentions. This paper is about computing an optimal sequence of stimulus in order to minimize the number of stimuli, hence improving the performance. To accomplish this, we model the problem as a partially observable Markov decision process (POMDP), which is a model for planning in partially observable stochastic environments. Through simulation and human subject experiments, we show that our approach achieves a significant performance improvement in terms of the success rate and the bit rate.

Author Keywords
P300, brain-computer interface (BCI), partially observable Markov decision process (POMDP)

ACM Classification Keywords
H.5.2 [Information Interfaces and Presentation (e.g., HCI)]: User Interfaces --- Brain-Computer Interfaces; I.2.8 [Artificial Intelligence]: Problem Solving, Control Methods, and Search --- Partially Observable Markov Decision Processes

General Terms
Algorithms, Design, Experimentation, Performance

INTRODUCTION
A brain-computer interface (BCI) aims to provide a communication channel for conveying messages and commands from the brain to the external system by interpreting brain activities [22]. There are a variety of devices and methods for BCI, including non-invasive methods such as using encephalography (EEG). The EEG-based BCI is perhaps the most popular method to date, because it is relatively easy and cheap to set up the system [15]. One of the most reliable signal features of EEG is the P300 evoked potential [14], which is a positive peak in the signal amplitude at about 300ms after a stimulus is given to the user’s attention [6, 22].

A number of BCI systems using P300 have been proposed in the literature, including the P300 speller [6]. In the P300 speller system, the user faces a 6x6 matrix of letters, and the user gazes at one of the 36 letters that one desires to select. The letters in a row or a column are flashed (stimulated) in a random order. If the letter that the user is gazing at flashes, P300 is generated at about 300ms later. We can thus train a classifier to detect this P300 to identify the desired letter. Some BCI systems with a smaller matrix may flash one letter at a time, e.g., [3].

Figure 1 shows a typical setup of EEG-based BCIs. These traditional systems generate flashes in a random order. For example, the P300 speller generates the same number of flashes for every row and column in a random order. We can note that however, by determining the optimal sequence of flashes, we can identify the desired selection from a smaller number of flashes. For example, based on the histo-
ry of flashes and the P300 detection results, if the probability that the first row contains the desired selection is very low compared to other rows, there is no reason to flash the first row. In contrast, if the probability is high for both the second and the third row, it is desirable to flash the second or the third row in order to resolve the uncertainty. Bell et al. [3] suggests that, since we can identify the most likely selection during flashes by maintaining a running sum of P300 classifier output, we may be able to reduce the number of flashes in identifying the desired selection.

This paper presents a systematic approach to finding an optimal sequence of flashes in order to identify the desired selection using the fewest possible number of flashes in EEG-based BCIs. Determining the best flash sequence based on the accumulated results of P300 detection is viewed as a sequential decision making problem, and we adopt the partially observable Markov decision process (POMDP) model [11] for the representation of the problem. The POMDP model provides a rigorous framework for representing sequential decision making problems under limited sensory capabilities of agents. It perfectly fits our purpose since we have to deal with P300 classifier errors. Although it is computationally infeasible to find an optimal solution from POMDPs, the recent development of fast approximate algorithms such as point-based value iteration (PBVI) [16], and heuristic search value iteration (HSV) [19], has made the POMDP approach practical for a wide variety of real-world applications such as spoken dialogue management [22] and assisted daily living [9].

Most of the previous works on EEG-based BCIs have been focused on the lower level of the interface. Whereas they are concerned with better feature extraction or classification methods for detecting single P300 from EEG while relying on a simple, random order of flashes, our focus is on finding an optimal sequence of flashes given a P300 classifier which is already implemented. Hence our work addresses the higher level of the interface, which is an important but currently missing part for an effective BCI.

BACKGROUND
In this section, we briefly review the facts about P300 in EEG and the common settings used in EEG-based BCIs which we will also be adopting in our work. We also review the standard definition of POMDPs for the sake of presenting our work.

**Electroencephalography (EEG) and P300 Interfaces**

EEG signals are the electrical signals recorded from the scalp and produced by the electrical activity of neurons in the brain. The electric potentials reflect the summation of the synchronous electrical activity of thousands or millions of neurons that are near the electrode for recording the EEG signals [20].

Event related potential (ERP) is elicited by an infrequent or particularly significant somatosensory stimulus. P300 is the positive peak component of ERP at about 300ms after the stimulus [6, 22]. P300 is known as one of the most reliable signals for composing BCI systems. However, the P300 elicited by the stimulus cannot be obtained easily, because EEG captures the brain activities from numerous sources and the P300 may be buried deep [8]. Fortunately, there exist several P300 feature extraction methods and classification methods for detecting the P300 component of ERP. The feature extraction methods include the averaging of EEG signals [8], the Mexican hat wavelet [8, 17], and the spatial filter algorithm [10]. Once the relevant features are extracted, classification methods such as Fisher’s linear discriminant, stepwise linear discriminant analysis (SWLDA), and support vector machine (SVM) [13] are used to detect the existence or absence of P300 in the EEG.

BCI systems based on P300 have a typical architecture as shown in Figure 2. There are components for stimulus generation, signal acquisition, preprocessing, and translation. The stimulus generator component gives stimuli to a user to elicit P300 on the desired situation. The signal acquisition component records the EEG signal for the given stimulus. The preprocessing component carries out feature extraction for detecting P300 from the given EEG signal. The translation component classifies the existence of P300, and sends appropriate commands to the external devices.

**Partially Observable Markov Decision Processes (POMDPs)**

A POMDP is a mathematical model for sequential decision making problems under uncertainty in the observation. It is defined by 8-tuple \((S, A, Z, b_0, T, O, R, \gamma)\): \(S\) is the set of environment states; \(A\) is the set of actions available to the agent; \(Z\) is the set of all possible observations; \(b_0\) is the initial belief where \(b_0(s)\) denotes the probability that the environment starts in state \(s\); \(T\) is the transition probability where \(T(s, a, s')\) denotes the probability that the environment changes from state \(s\) to state \(s'\) when executing action \(a\); \(O\) is the observation probability where \(O(s, a, z)\) denotes the probability that the agent makes observation \(z\) when executing action \(a\) and arriving at state \(s\); \(R\) is the reward function where \(R(s, a)\) denotes the reward received by the agent when executing action \(a\) in state \(s\); \(\gamma\) is the discount factor such that \(0 \leq \gamma \leq 1\).
Since we assume that the agent cannot directly know the environment state, it maintains the probability distribution of the states based on the history of observations and actions. The probability distribution is defined as a belief state $b$ where $b(t)$ denotes the probability that the state is $s$ at time step $t$. The belief state $b_t$ can be regarded as the posterior distribution of states given the initial belief $b_0$ and the history $\{a_0, z_1, a_1, z_2, \ldots, a_{t-1}, z_t\}$:

$$b_t(s) = P(S_t = s | a_0, z_1, a_1, z_2, \ldots, a_{t-1}, z_t)$$

Upon execution action $a_t$ and making observation $z_{t+1}$ in belief state $b_t$, the belief state $b_{t+1} = \tau(b_t, a_t, z_{t+1})$ at the next time-step is computed by the Bayes rule,

$$b_{t+1}(s') = \frac{O(s', a_t, z_{t+1}) \sum_{s \in S} T(s, a_t, s') b_t(s)}{P(z_{t+1}|b_t, a_t)},$$

where $P(z_{t+1}|b_t, a_t)$ is the normalizing constant such that $\sum_s b_{t+1}(s) = 1$.

A policy determines the actions to be executed by the agent. Specifically, a policy $\pi$ of a POMDP can be defined as a mapping from belief states to actions, i.e., $\pi: \Delta S \rightarrow A$. Every policy has an associated value function, which is (in the case of infinite horizon problems) the expected cumulative discounted reward by following the policy starting from a given belief state. When solving a POMDP, we search for an optimal policy that maximizes the value for each belief state. The maximum value for the belief state can be computed recursively

$$V^*(b) = \max_a \left[ R(b, a) + \gamma \sum_z P(z|b, a) V^*(\tau(b, a, z)) \right],$$

where $P(z|b, a) = \sum_{s'} O(s', a, z) \sum_{s} T(s, a, s') b(s)$ and $R(b, a) = \sum_{s} R(s, a) b(s)$. The optimal value function $V^*$ can be obtained by a series of dynamic programming back-up

$$V_t(b) = HV_{t-1}(b)$$

$$= \max_a \left[ R(b, a) + \gamma \sum_{z \in Z} P(z|b, a) V_{t-1}(\tau(b, a, z)) \right],$$

for every belief state $b \in \Delta S$. We can also derive that value function $V_t$ is piecewise linear and convex, hence it is represented as a set of $\alpha$-vectors $\Gamma_t = \{\alpha_1, \ldots, \alpha_m\}$ and the value at a particular belief state $b$ is calculated as

$$V_t(b) = \max_{a \in \Gamma_t} \sum_{s \in S} \alpha(s) b(s).$$

Once we compute the optimal value function $V^*$, the optimal policy is obtained by

$$\pi^*(b) = \arg \max_a \left[ R(b, a) + \gamma \sum_z P(z|b, a) V^*(\tau(b, a, z)) \right].$$

Since there are infinitely many belief states, it is intractable to compute the optimal value function and the optimal policy. Some of the POMDP algorithms such as the witness algorithm [11] exploit the piecewise linear and convex property of value functions, but they are still limited to problems of small sizes. Instead, we settle for approximate algorithms such as point-based value iteration (PBVI) [16] or heuristic search value iteration (HSVI) [19], which focus the computational effort around the reachable belief states. These approximate algorithms are scalable, yet the solutions found are almost optimal in various benchmark POMDP problems. A complete review of exact and approximate algorithms for POMDPs is outside the scope of this paper, so we refer the readers to the references mentioned above.

**SYSTEM ARCHITECTURE**

Figure 3 shows the architecture of our BCI system. It is similar to the system described in Bell et al. [3], except that our system uses the POMDP policy for determining which letter to flash (stimulate).

The flash scheme of our system follows the paradigm similar to that of the P300 speller system: The user gazes at the target letter and the letters in the matrix are flashed in 250ms intervals, one letter at a time. Each flash turns on the letter for 125ms and turns off for another 125ms. A test consists of a series of flashes for identifying the target letter, and a pause interval of 2.5s is given between consecutive tests.

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Preprocessor

In order to construct the preprocessor and the classifier, we first prepare the training data consisting of P300 epoch instances each labeled either a target or a non-target. This training data is gathered using the same flash scheme outlined earlier in this section.

Since the raw signals contain a significant amount of high frequency noise, they are band-pass filtered (0.5-30Hz) with a 6th order Butterworth filter and down-sampled to 100Hz. We then extract features from the epoch data using the spatial projection algorithm [10]. This algorithm generates a set of filters which maximally discriminates between target and non-target epoch instances in the training data. More formally, given \( m \) channels and \( n \) time points of an epoch, the spatial projection algorithm generates a maximum of \( m \) linear filters. Each filter linearly transforms the epoch data from multi-channel (i.e. epoch) to one virtual channel. Each filter \( f_j \) (an \( m \)-dimensional column vector) is said to have discriminative power \( d_j \), which is the value of Fisher criterion for linearly separating the target and the non-target instances in the training data. Given an epoch instance \( E_i \) (an \( m \times n \) matrix) for the \( j \)-th flash, the feature vector of the epoch using filter \( f_j \) is computed by \( f_j^T E_i \) (an \( n \)-dimensional row vector). The exact number of filters is usually determined by cross-validation. In our case, we limit the maximum number of filters to 5, since this number was sufficiently large for most of the subjects participating in this study and guaranteed timely processing of data for the computational resource available.

Classifier

To identify whether an epoch instance contains P300, we trained a classifier on the preprocessed feature vectors using the LIBLINEAR package [5]. Specifically, we used the L2-regularized logistic regression to map the binary output to a real value between 0 and 1, which represents the posterior probability that the feature vector is a target (i.e., the epoch contains P300). The parameters of the classifier are decided by 5-fold cross-validation on training data.

POMDP PLANNER

The POMDP planner plays the central part of our BCI system. In order to compute the optimal sequence of flashes using POMDP, we first need to model the problem. Once the modeling is done, we use a POMDP algorithm to obtain an optimal policy, which will determine the flash sequence. However, our approach is more sophisticated than plain models and algorithms since we have to take into account some important constraints of the system, such as the delay in the P300 (the relevant observation is not available throughout the next two flashes) and the repetition blindness (P300 may not be elicited if two flashes is given on a target letter within 500ms).

POMDP Modeling of BCI

Our problem in hand is to find the target letter as accurately as possible while using as a small number of flashes as possible. Conceptually, this problem can be considered as an extension of the tiger problem in the POMDP literature [11], where the number of doors matches the number of letters in the stimulus matrix. The exact description is given below.

Let \( N \) be the number of letters in the matrix. In this paper, we consider \([2x2]\) and \([2x3]\) matrices, hence \( N = 4 \) and \( N = 6 \), respectively. The states in the POMDP correspond to the target letters, hence a total of \( N \) states. For each letter in the matrix, we can either flash it in the hope of detecting P300 (\( N \) flash actions) or claim that it’s the target letter (\( N \) select actions), hence a total of \( 2N \) actions. The output value from the P300 classifier serves as the observation, where the real value between 0 and 1 is discretized into intervals of size 0.1 (e.g., \( z_1 \) for the output value in \([0.0, 0.1)\), \( z_2 \) for the output value in \([0.1,0.2)\), etc.), hence a total of 10 observations.

To make the system identify the target letter as soon as possible, we give -0.1 reward for the flash actions, +1 reward for the select actions that make a correct claim of the target letter, and -10 rewards for the select actions that incorrectly make a claim on a non-target letter.

We define the transition probability for each flash action as an identity matrix: we assume that the target does not change to some other letter within a test. Hence, we assign the transition probability of 1 if the state at the current time step is the same as the state at the next time-step, and 0 if the state at the current time-step is different from the state at the next time-step. For select actions, the transition prob-
abilities are uniform: we assume that the target letter will reset to any letter in the matrix with the same probabilities between consecutive tests.

The errors in the P300 classification results are modeled as observation probabilities. Specifically, we assume that the classifier output follows the beta distribution when flashing the target letter. The parameters $\alpha$ and $\beta$ of the beta distribution are obtained from the training data. We also assume that the distribution of output values when flashing a non-target letter is symmetric to the case when flashing the target letter. Hence, if $\alpha_{\text{target}}$ and $\beta_{\text{target}}$ are the parameter values of the beta distribution for flashing the target letter, we set $\alpha_{\text{non-target}} = \beta_{\text{target}}$ and $\beta_{\text{non-target}} = \alpha_{\text{target}}$ for the beta distribution for flashing a non-target letter. Since these parameter values are different among the subjects, and POMDP algorithms take hours to find the optimal policy, we set the observation probability to 9 (and 11) different beta distributions that can appear in the human experiments on the system with $[2 \times 2]$ (and $[2 \times 3]$) matrix. Hence, we have 9 (and 11) POMDP models with different observation probabilities for $[2 \times 2]$ (and $[2 \times 3]$). The discount rate is set to 0.99, and the initial belief is set to the uniform distribution. This completes the specification of the POMDP model.

We briefly summarize how the model works. The state corresponds to the target letter in the current test, of which the BCI system does not have the direct knowledge. Hence, the system has to infer which letter is the target by some sensor. Since the BCI system does not have the direct knowledge, the observation is available at all time-steps (in our case, sequence of length 2) during each dynamic programming backup, in contrast to finding the best single action in standard POMDPs without delayed observations.

Repetition blindness

The repetition blindness refers to the situation where P300 may not be elicited when two flashes on the target letter are given within 500ms [7, 12]. For example, when the target letter “A” is flashed and the “A” is flashed again within 500ms, the EEG signal corresponding to the second flash may not contain P300. A simple way to avoid this phenomenon is to make sure that the flash is not given on the same letter within 500ms. Since our flash scheme makes two flashes in 500ms, the flash at the current time-step should be different from the previous two flashes. In terms of our POMDP model, the action at the current time-step should be different from the previous two actions.

This constraint can also be handled by modifying the standard dynamic programming backup operation in POMDP algorithms: when we compute the best action that yields the best value, we only consider the actions that were not executed in the previous two time-steps.

POMDP algorithm for BCI

We now present our implementation of POMDP algorithm that addresses the P300 delay and the repetition blindness.

Basically our implementation is a modified version of the PBVI algorithm [16], which we refer to as BCI-PBVI. Figure 5 shows the main loop of the algorithm. As in PBVI, this algorithm requires the set $B$ of randomly sampled belief states (i.e., belief set) for restricting the dynamic programming backup to those belief states (i.e., point-based

| Algorithm | BCI-PBVI($B, K, D, \epsilon$) |
| INPUTS: | belief set $B$; attentional blink length $K$; observation delay $D$; required precision $\epsilon$ |
| for all | action sequence $\langle a_1, \ldots, a_D \rangle$ of length $D$ do |
| Initialize $\Gamma_{a_1,\ldots,a_D} = \{0\}$ |
| end for |
| repeat |
| for all | action sequence $\langle a_1, \ldots, a_D \rangle$ of length $D$ do |
| $\Gamma'_{a_1,\ldots,a_D} = \text{backup}(B, K, D, \Gamma, a_1, \ldots, a_D)$ |
| end for |
| $\delta = \text{difference}(B, \Gamma, \Gamma')$ |
| $\Gamma = \Gamma'$ |
| until $\delta < \epsilon$ |
| return $\Gamma$ |

Figure 5. Top-level pseudocode of BCI-PBVI.
Algorithm backup($B, K, D, \Gamma^{t-1}, a_1, \ldots, a_D$)

INPUTS: belief set $B$; attentional blink length $K$; observation delay $D$; set of $\alpha$-vectors for $(t-1)$-step value function $\Gamma^{t-1}$; the action sequence of interest $(a_1, \ldots, a_D)$

$A_{allow} = A - \{a_{D-K+1}, \ldots, a_D\}$

for all action $a \in A_{allow}$ do
  for all observation $z_D \in Z$ do
    for all $\alpha$-vector $a_i \in \Gamma^{t-1}_{a_{2-D}, a_D, a}$ do
      $a_i^{a, z_D}(s) = \gamma \sum_{s'} T(s, a_i, s') O(s', a_i, z_D) a_i(s'), \forall s \in S$
    end for
    $\Gamma^{t, a, z_D}_{a_{1-D}} = \bigcup_{a_i} \{a_i^{a, z_D}\}$
  end for
end for

for all belief state $b \in B$ do
  for all action $a \in A_{allow}$ do
    $a_b^{a} = \sum_{s_1, \ldots, s_D \in S} T(s_1, a_1, s_2) \cdots T(s_{D-1}, a_D, s_D) R(s_D, a) + \sum_{s \in S} \arg\max_{a \in A_{allow}} \sum_{s \in S} b(s) a_b^{a}(s)$
  end for
  $a_b = \arg\max_{a \in A_{allow}} \sum_{s \in S} b(s) a_b^{a}(s)$
  $\Gamma^{t}_{a_{1-D}} = \Gamma^{t}_{a_{1-D}} \cup \{a_b\}$
end for
return $\Gamma^{t}_{a_{1-D}}$

Figure 6. The backup operator for BCI-PBVI.

backup). Whereas the standard PBVI maintains the best $\alpha$-vector and the corresponding best action for each sampled belief, the BCI-PBVI maintains $\alpha$-vector for all length-$D$ action sequences for each sampled belief state. It is like computing the action-value function rather than the state-value function. This is necessary in order to prevent the same action being executed within $K$ time-steps, i.e., handle repetition blindness.

For every possible length-$D$ action sequence, we update the corresponding set of $\alpha$-vectors using the backup procedure. Figure 6 shows the pseudocode of the procedure, and it is the central part of our algorithm. Assuming that the current time-step is $T$, we first compute the set $A_{allow}$ of actions that are allowed to execute at time-step $T + D$, i.e., the set of actions except those appearing in the last $K$ steps in the sequence. Among the set of $\alpha$-vectors that are computed in the previous iteration, only those with action sequences ending with allowed actions are valid $\alpha$-vectors for the backup. The first loop in the pseudocode carries out this task. The second loop performs the actual point-based backup on the belief set $B$. For action sequence $a_1, \ldots, a_D$, and a belief state $b$, we compute $a_b$ such that $\sum_{s} b(s) a_b(s)$ is the maximum expected value gathered after $D$-steps in the future, if, starting from the current belief state $b$, we execute the action sequence $a_1, \ldots, a_D$. Cautious readers may question that $a_b$ should also consider the rewards gathered within $D$-steps in the future, but as we will see shortly, it is not necessary to do so.

Belief update
Assume that, at time-step $t$, we have executed an action sequence $a_{t-D}, \ldots, a_{t-1}$, and deciding which action $a_t$ to execute. According to the definition of $\alpha$-vectors in BCI-PBVI, we cannot use the current belief state $b_t$. Instead, we need the belief state $b_{t-D}$ of $D$-steps in the past, and this is the belief state we maintain while executing the policy.

Once we execute $a_t$ and observe $z_t$, the belief state $b_{t-D}$ is updated by

$$b_{t-D+1}(s') = \frac{O(s', a_{t-D}, z_t) \sum_s T(s, a_{t-D}, s') b_{t-D}(s)}{P(z_t | b_{t-D}, a_{t-D})}$$

where $P(z_t | b_{t-D}, a_{t-D})$ is the normalizing constant. Hence, in order to perform appropriate belief update, we need to remember the past belief state $b_{t-D}$ as well as the action sequence $a_{t-D}, \ldots, a_{t-1}$.

Optimal action selection
At this point, selecting the optimal action for execution is quite straightforward. Assume once again that, at time-step $t$, we have executed an action sequence $a_{t-D}, \ldots, a_{t-1}$, and deciding on which action $a_t$ to execute. Since we have the belief state $b_{t-D}$ at hand, we compute $\sum_s b_{t-D}(s) a(s)$ for

$$\forall a \in \Gamma^{t}_{a_{t-D}, \ldots, a_{t-1}}$$

and execute the best action associated with the $\alpha$-vector that yields the best value.
SIMULATION EXPERIMENTS

Methods

Baseline
The baseline method follows a flash sequence decided by random order and hence it is equivalent to the method used in traditional BCI systems including Bell et al. [3]: the flash sequence is randomized among the letters that are not flashed within the current trial, where the trial refers to a subsequence of length equal to the letters in the matrix, hence each letter is flashed once per trial. However, our baseline method additionally takes repetition blindness into account: if a letter was flashed within 500ms, it will not be considered as a candidate for the current flash. The decision is made by the total score from the classifier: the output value of the classifier is regarded as the posterior probability, and the letter with the largest sum throughout trials is selected as the target letter. Hence, the method can be stopped at the end of any trial during a test, and determine the most likely target letter. The total score of a letter can be regarded as the posterior probability of being the target letter given the history of flashes and classifier output values. Note that the baseline method has no explicit “stop and select” decision making mechanism.

POMDP with select actions (PWSA)
The PWSA method uses the optimal flash policy computed from POMDP. The select actions represent the explicit decision making mechanism.

POMDP without select actions (PWOSA)
The PWOSA method uses the same set of actions as PWSA except the select actions. When stopped, we determine the target letter with the highest belief state probability. We prepared this method for the sake of performance comparison with the baseline method when the same number of flashes is used. Ideally, this method will be more efficient than the baseline method in terms of the number of flashes because the flash sequence is determined by the POMDP policy, rather than by some random distribution. Conceptually, this method can be considered as an optimal policy from a POMDP model with negative infinite rewards for selecting an incorrect target letter.

Setup
We performed 20 simulations on [2x2] and [2x3] matrices, each simulation consisting of 10000 tests. Output values from the classifier (i.e. observations) are sampled from the beta distribution with parameters $\alpha_{\text{target}} = \beta_{\text{non-target}} = 1.228$ and $\beta_{\text{target}} = \alpha_{\text{non-target}} = 0.625$, which were obtained from the pilot experiment involving one of the human subjects.

Each test consists of 40 flashes for the [2x2] matrix and 60 for the [2x3] matrix. For the baseline method, these numbers correspond to 10 observations for each letter in the matrix, whereas the PWSA and PWOSA methods may have different number of observations for each letter. Note also that the baseline and PWOSA methods run until the test ends, whereas the PWSA method can terminate early when the final select action is executed.

Measurements
We measured the performance of each method using the success rate and the bit rate.

Success rate
The success rate is defined by the percentage of tests in the simulation with correct identification of the target letters. We stopped the methods at the end of each trial (4 flashes for [2x2] and 6 flashes for [2x3]), and measured the success rate. Note that the PWSA method can terminate before the specified number of flashes. In this case, we extended the results until the end of tests. For example, if the PWSA method terminated during the 5th trial with an incorrect target letter, the method is regarded as selecting an incorrect target letter for all subsequent trials, and vice versa.

For all three methods, if the method has the same total score or the same maximum belief value for $K$ letters, we gave a partial success of $1/K$.

Bit rate
The bit rate represents the quantity of transferred information per unit time during communication [18, 21]. The bit rate is defined as $B \cdot D$, where $B$ is the number of bits per decision and $D$ is the number of decisions per unit time. Let $N$ be the total number of letters in the matrix and $P$ be the success rate. Then we have

$$B = \log_2 N + P \log_2 P + (1 - P) \log_2 \frac{1 - P}{N - 1}.$$ 

In order to measure $D$, we used the following scheme: For the baseline method, since the decision has to be delayed (260ms) until the result of the last flash is available, we add the delay to the time spent on flashes. For example, if we have flashed 40 times, then the total time spent for the decision is calculated as 250ms * 40 flashes + 260ms. For the PWSA and PWOSA methods, since it takes an additional 115ms delay for updating the belief state, we add a delay of 375ms to the time spent on flashes. For example, if we have flashed 40 times, the total time for the decision is 250ms * 40 flashes + 375ms. $D$ is calculated as the reciprocal of the total time.

Since the bit rate changes depending on the success rate, we calculated the bit rates for different success rates. The tradeoff here is that we can improve the success rate by increasing the number of flashes, but more time is accordingly spent per decision. For the PWSA method, since it has its own explicit termination mechanism, we will report only one value for the bit rate measurement.

Results
Figure 8 shows the success rate results for the three methods. The performance gap between the baseline method
and the PWSA/PWOSA method is larger in the [2x3] matrix than in the [2x2] matrix. We conjecture that the performance gap will become even larger when we experiment on larger stimulus matrices. The baseline and the PWOSA methods will converge to a 100% success rate as the number of flashes goes to infinity. In contrast, the success rate of the PWSA method doesn’t due to the bias inherent in the reward function. However, having an infinite number of flashes is not a practical assumption, and this kind of bias is necessary if we ever want the method to have some explicit termination mechanism. In our experiments, the PWSA method converges to a success rate close to 100% quickly when forced not to terminate before the specified number of flashes, which is sufficient to demonstrate the validity of our approach. Figure 7 shows the bit rate results. The PWOSA method is always significantly better than the baseline method. The PWSA method yields the success rates of 0.980 for [2x2] and 0.977 for [2x3], which correspond to 23.840 bits/min and 23.080 bits/min respectively. Table 1 summarizes the bit rate results of the three methods.

Comparing the bit rates at the best achievable success rates, the PWSA method improves the bit rates by 220% to 249%. Comparing to the best bit rates achievable by the baseline method, the PWSA method achieves improvements of 126% to 158%.

1 The baseline and PWOSA methods can achieve higher bit rates by lowering the success rate, but we set the minimum to 75% since we also want a sufficiently high success rate.

HUMAN SUBJECT EXPERIMENTS

Subjects and Experimental Setup

Nine (9) able-bodied male students at the Korea Advanced Institute of Science and Technology (KAIST) participated in the human subject experiments.

We compared the performances of the baseline and PWSA methods on the [2x2] and [2x3] matrices for these human subjects. Each test randomly assigned a target letter, while making sure that each letter was selected as the target letter exactly 10 times for the [2x2] matrix and 5 times for the [2x3] matrix. Hence, we performed a total of 40 tests for the [2x2] matrix and 30 tests for the [2x3] matrix. As in the simulation experiments, each test consisted of 40 flashes for the [2x2] matrix and 60 flashes for the [2x3] matrix.

Procedure

We first prepared a number of different POMDP models with varying observation probabilities, since the classifier showed different error rates depending on the human subject. By varying the parameters of the beta distribution, we obtained 9 different models of the [2x2] matrix, and 11 for...
the [2x3] matrix. We pre-computed the optimal policy for each model, since our implementation of the POMDP algorithm currently takes hours to finish. This is due to the fact that the point-based backup requires enumerating all possible action sequences of length $D$. Further optimization via pruning useless action sequences is left as a future work. We used 1028 randomly selected belief states for the [2x2] matrix, and 1030 for the [2x3] matrix.

At the onset of the experiment for each subject, we carried out a short pilot experiment where we gathered the training data for the preprocessor/classifier. Once they were trained, we performed cross-validation evaluation, chose the POMDP model with the minimum KL-divergence, and used the corresponding optimal POMDP policy.

We measured the success rates and the bit rates for comparing the performances of the baseline and PWSA methods.

**Results**

We originally involved nine (9) human subjects, but two (2) of them had beta distributions very far from any of the pre-computed models. Hence we use the data from seven (7) human subjects.

Figure 10 shows the success rate results of the human experiments. The top graph shows the result on [2x2] matrix, and the bottom graph shows the result on [2x3] matrix.

![Success Rate Results](image)

Figure 9 shows the bit rate results. The PWSA method yielded an average success rate of 98.2% for the [2x2] matrix and 97.6% for the [2x3] matrix. The corresponding bit rate is 24.368 bits/min for the [2x2] matrix, and 21.367 bits/min for the [2x3] matrix. In the case of the baseline method, we can control the success rate by changing the number of flashes, and the maximum average success rates are 96.4% for the [2x2] matrix and 92.9% for the [2x3] matrix. Regardless of how we set the success rate for the baseline method, the PWSA yielded higher bit rates. The summary of the bit rate results is shown in Table 1. Compared to the bit rates at the best achievable success rates, the PWSA method improves the bit rates by 242% to 265%. Compared to the best bit rates achievable by the baseline method, the PWSA method achieves improvements of 135% to 151%. These results are similar to those from the simulation experiments.

**DISCUSSION**

In this paper, we have presented a P300-based BCI system that uses POMDP for calculating the optimal flash sequence. In contrast to the previous body of research that concentrate on obtaining better feature extraction and classification algorithm from the raw EEG signals, our work provides a unified framework for building the BCI system. Bell et al. [3] have roughly suggested the idea of using the confidence values from the P300 classifier for optimizing the flash sequence, but to the best of our knowledge, our work is the first to address the problem in a principled way.
The contributions of this paper are as follows: First, we provided a formal decision-making model for P300-based BCI. Specifically, we showed how the POMDP model with observation delays can be adapted to BCI. Although we explained in the context of P300-based BCI systems, we believe that our approach is general enough to be easily applied to other BCI paradigms. Second, we presented a novel point-based algorithm for solving POMDPs with observation delays. The algorithm extends the standard point-based backup operator to handle observation delays. Third, we report experimental results using simulation as well as human subjects. Our POMDP-based method achieves significant performance improvement over the baseline method currently used in other BCI systems.

Currently, we are working on improving the speed of the algorithm for POMDPs with observation delays. One of the most time-consuming aspects of our algorithm is in the enumeration of all possible action sequence of length equal to the delay. Since some action sequences may be inferior to others, a combination of forward search and dynamic programming may yield substantial improvement in the speed. We are also working on applying the technique to P300 speller, where the user intentions exhibit more regularity. We strongly believe that we can achieve a magnitude of order improvement in performance (bit rate) if we embed the bigram/trigram model of alphabets into the intention-level transition probability of the POMDP. Finally, we are investigating into the methods [4] that enable the adaption of model to individual subjects without explicit pilot trial experiments or pre-computing optimal policies by enumerating candidate models.

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